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METHODS OF STANDARDIZING ICE CREAM MIXES

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"Standardizing" an ice cream mix means calculating the relative amounts of ingredients required for a finished product of a desired composition. The calculations are made for a definite weight of ice cream mix, the exact amount depending upon the size of the standard batch processed in a given plant. For the sake of simplicity, however, the calculations made here in most cases will be for a 100-pound batch, since that amount offers a satisfactory basis for making any desired quantity of mix.

Errors often occur in making mixes. These are commonly due to careless weighing of products, or to the fact that the composition of the ingredients are different from the assumed values used during the original calculations. Provided the errors are great enough, a recalculation is necessary, to ascertain how the original mixture must be modified to assure the desired composition. This process is termed "restandardization."

A satisfactory mix for plain as well as for fruit ice creams contains 12.0 per cent fat, 10.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and sufficient gelatin for proper stabilization. In California,[2] the minimum legal fat content of vanilla ice cream is 10.0 per cent; of fruit ice cream, 8.0 per cent. The fat contained in 100 pounds of legal vanilla ice cream would satisfy the requirements for 125 pounds of fruit ice cream —which equals a dilution of 25 per cent. If a higher dilution is desired, it would be necessary for the basic mix to contain more than 10 per cent fat. The standard adopted in many cases is considerably higher than the legal requirements.

[1] Associate in Dairy Industry.

[2] Agricultural Code of California, effective August 21, 1933.

This circular is intended to aid commercial ice cream manufacturers in making the computations necessary for the proportioning of ingredients in ice cream mixes. Not all the available methods for standardization and restandardization are considered in this publication; but there are definite reasons for including the methods given as can be seen from the brief advance discussion that follows.

1. Ordinarily the *Pearson square method* is as simple as any. With this method no formulas need be memorized. The computations, however, are comparatively long, particularly when used to standardize a complex mix such as one requiring the use of evaporated or sweetened condensed milk.

2. The *serum point* or *formula method* is shorter than the Pearson square. It can be used in practically all cases and does not become unduly complicated when used for the solution of complex mixes. Its formulas appear at first sight to be rather complicated, but are easily learned.

3. The main advantages of the *algebraic method* are accuracy and general applicability. In most cases, however, it is longer than the serum point method and, as ordinarily used, is scarcely more accurate; besides, it is often more difficult to check if an error is introduced in the calculations.

4. The *graphic method,* although not so generally used, is fairly simple and is accurate enough for practical purposes. Its main advantage seems to lie in its easy application to restandardization problems. Those accustomed to the use of graphs should find it convenient and satisfactory.

5. *Tabulated recipes* (table 2) for ice cream and ice milk[3] on a 100-pound basis, will be sufficient to meet the needs of many ice cream makers.

The particular method chosen for use in *restandardization* will depend upon the type of problem under consideration. Examples are given in a later section to illustrate the application of several methods.

[3] Ice milk is a frozen dairy product similar in composition to ice cream except that its percentage fat content is usually much lower—the minimum legal standard in California being 4 per cent fat.

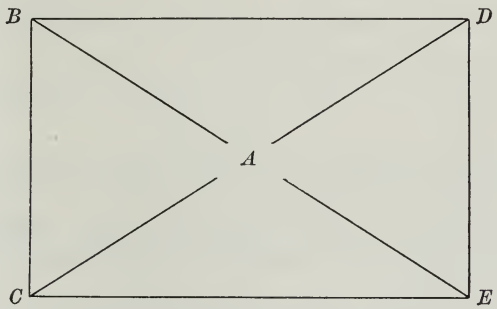
METHODS OF MIX STANDARDIZATION

PEARSON SQUARE METHOD

The Pearson square method, rather generally used in standardizing milk and cream for fat content, will serve equally well in standardizing mixes for milk fat and for milk-solids-not-fat. Here, however, one must make certain arbitrary divisions of the mix to facilitate calculation.

In this method, a square or rectangle serves as a guide to the necessary steps in calculating the relative amounts of two ingredients required for a mixture of desired composition. Obviously the percentage composition of one ingredient must be above and that of the other must be below the desired value in the final mixture.

Before proceeding to a consideration of its application to specific problems, the following working diagram is given to illustrate the basis of this method.



A = Percentage of the desired constituent in the final mixture (e.g., milk fat or milk-solids-not-fat)

B = Percentage of this constituent in one of the ingredients

C = Percentage of this constituent in the other ingredient

D = Unit parts of the ingredient in the final mixture containing the B percentage. D is a positive number and equals the difference between C and A

E = Unit parts of the ingredient in the final mixture containing the C percentage. E is a positive number and equals the difference between A and B

The total unit parts of the final mixture having A percentage of the desired constituent equals the sum of D and E. Consequently,

$$\left.\begin{array}{l}\text{the fraction of the total mixture which will be the ingredient with } B \text{ percentage}\end{array}\right\} = \frac{D}{D+E}, \quad \text{and}$$

$$\left.\begin{array}{l}\text{the weight of the ingredient with B percentage}\end{array}\right\} = \left[\frac{D}{D+E}\right] \times \left[\begin{array}{c}\text{total weight of} \\ \text{the desired mixture}\end{array}\right]$$

Likewise,

$$\left.\begin{array}{l}\text{the fraction of the total mixture which will be the ingredient with } C \text{ percentage}\end{array}\right\} = \frac{E}{D+E}, \quad \text{and}$$

$$\left.\begin{array}{l}\text{the weight of the ingredient with } C \text{ percentage}\end{array}\right\} = \left[\frac{E}{D+E}\right] \times \left[\begin{array}{c}\text{total weight of} \\ \text{the desired mixture}\end{array}\right]$$

The solution to the following problem illustrates the use of this method in standardizing for milk-solids-not-fat and for milk fat. In all problems of this type it is necessary to first standardize for the milk-solids-not-fat in the serum of the mix. Even though there are more than two sources of milk-solids-not-fat the products can ordinarily be grouped so that standardization is accomplished by considering only two sources. The reason for this is the fact that the skim-milk portions of both milk and cream are "normal" skim milk and for practical purposes both may be considered to contain the same percentages of milk-solids-not-fat.

Problem: Suppose it is desired to prepare 100 pounds of mix containing 12.0 per cent fat, 10.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.5 per cent gelatin. Consider the following products to be available: cream testing 40.0 per cent fat; milk testing 4.0 per cent fat; condensed skim milk containing 30.0 per cent milk-solids-not-fat; and the nonmilk products, sugar and gelatin. The steps in the calculations are given below and the results are verified in tabulated check 1 on page 6.

Since the nonmilk products can be calculated directly, their weights should first be subtracted from the total weight of the mix. Thus, 100 pounds of mix minus 15.5 pounds of sugar and gelatin leaves 84.5 pounds of milk products, which, in this case, will be standardized by the Pearson square method.

By temporarily eliminating from consideration the pounds of fat in the mix, the first part of the problem resolves itself into determining the percentage of milk-solids-not-fat in the serum and then using this value in the Pearson square as indicated below. Thus, 84.5 pounds of milk products minus 12.0 pounds of milk fat equals 72.5 pounds of serum in the mix. The average percentage of milk-solids-not-fat in the serum of the mix is calculated as follows:

$$\begin{array}{l}\text{Average percentage of milk-solids-not-fat in the serum of the mix}\end{array} = \frac{\begin{array}{c}\text{weight of milk-solids-not-fat} \\ \text{in the serum of the mix}\end{array}}{\text{weight of serum of the mix}} \times 100.$$

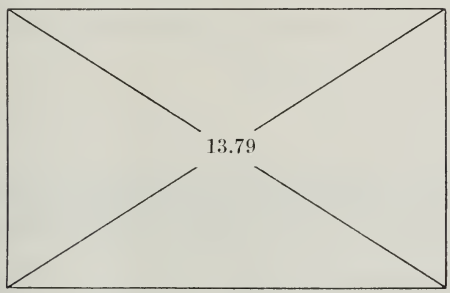
Hence,

$$\frac{10.0}{72.5} \times 100 = 13.79 = \begin{array}{l}\text{the average percentage of milk-solids-not fat in} \\ \text{the serum of the mix.}\end{array}$$

These calculations obviously indicate that the serum of the mix must

contain 13.79 per cent of milk-solids-not-fat. The products available from which to supply the serum of the mix with its required percentage of serum solids are: condensed skim milk testing 30.0 per cent milk-solids-not-fat, and the skim-milk portion of the milk and cream used. The latter can for all practical purposes be considered to contain 9.0 per cent serum solids. If in any specific case the composition of the skim milk deviates from this value, the correct figure can be substituted in the calculations. Thus, if the skim milk contained 8.8 per cent milk-solids-not-fat use 0.088 instead of 0.09. (For other values, see table 3.)

Follow the procedure already outlined on pages 3 and 4, but substitute numbers for the letters used in the general example. Thus,



4.79 = unit parts of condensed skim milk

16.21 = unit parts of skim milk from milk and cream

Then, $4.79 + 16.21 = 21.00$ = the unit parts of the mixture testing 13.79 per cent serum solids.

Hence, $\frac{4.79}{21.00} \times 72.5 = 16.54$ = pounds of condensed skim milk required.

Similarly, the pounds of skim milk could be calculated, but this is supplied by the milk and cream and is called "skim milk" only for purposes of computation. It should be apparent, therefore, that if the milk and cream are correctly proportioned they will of necessity contain the required amount of skim milk. Hence, the next calculations will deal with the proportioning of the milk and cream so as to supply the milk-fat requirements of the mix.

This can be accomplished by taking into consideration the milk fat of the mix which was temporarily eliminated in the previous calculations. The average percentage of fat in the milk and cream must be evaluated and used in the rectangle as indicated below. Thus,

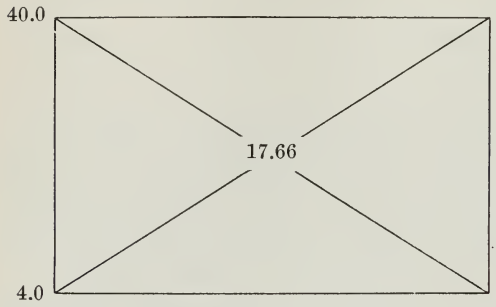
$$84.50 - 16.54 = 67.96 = \text{pounds of milk and cream desired.}$$

The average per cent of fat in the milk and cream is calculated as follows:

$$\text{Average per cent fat in the milk and cream} = \frac{\text{weight of fat in the milk and cream}}{\text{weight of milk and cream}} \times 100.$$

Hence, $\frac{12.0}{67.96} \times 100 = 17.66$ per cent fat in the milk-and-cream mixture.

Again use the Pearson square method, and calculate the weight of milk and cream required. Thus,



13.66 = unit parts of cream testing 40.0 per cent fat

22.34 = unit parts of milk testing 4.0 per cent fat

Then, $13.66 + 22.34 = 36.00$ = unit parts of mixture containing 17.66 per cent fat;

$\frac{13.66}{36.00} \times 67.96 = 25.79$ = pounds of cream; and,

$\frac{22.34}{36.00} \times 67.96 = 42.17$ = pounds of milk.

The weight of milk can also be calculated by subtracting the weight of cream already determined from the total weight of milk and cream. Thus,

$67.96 - 25.79 = 42.17$ = pounds of milk.

Finally the weights of sugar and gelatin are calculated by multiplying the weight of the mix by the fraction of each desired. Thus,

$100 \times 0.15 = 15.0$ = pounds of sugar; and,
$100 \times 0.005 = 0.5$ = pounds of gelatin.

The foregoing calculations are verified in tabulated check 1.

TABULATED CHECK 1

Ingredients used	Weight	Fat supplied	Milk-solids-not-fat supplied	Total solids supplied
	pounds	*pounds*	*pounds*	*pounds*
Condensed skim milk testing 30.0 per cent milk-solids-not-fat	16.54		4.96	4.96
Cream testing 40.0 per cent fat	25.79	10.32	1.39	11.71
Milk testing 4.0 per cent fat	42.17	1.69	3.64	5.33
Sugar	15.00			15.00
Gelatin	0.50			0.50
Totals supplied	100.00	12.01	9.99	37.50
Totals required	100.00	12.00	10.00	37.50

SERUM POINT OR FORMULA METHOD

It is possible by simple algebra to set up an equation for milk-solids-not-fat and one for the milk fat of the mix and from these derive general working formulas for the solution of most standardization problems. A set of such formulas, without showing derivation, is given below. In these formulas, the composition of skim milk, or the skim-milk portion of milk and cream, is taken as 9.0 per cent milk-solids-not-fat, which, as has already been explained, represents a safe average figure.

$$(1)\quad Z = \frac{R - 0.09S}{d - 0.09e}$$

$$(2)\quad X = \frac{(F - cZ) - bP}{a - b}$$

$$(3)\quad Y = P - X$$

In the above formulas,

X = pounds of cream with a fraction fat
Y = pounds of milk with b fraction fat
Z[4] = pounds of serum-solids concentrate with c fraction fat, d fraction milk-solids-not-fat, and e fraction serum
S = pounds of serum in the mix (milk products minus milk fat) containing R pounds of serum solids
F = pounds of milk fat in the mix
P = pounds of milk and cream, that is, the quantity $(X + Y)$

It may clarify these formulas somewhat to represent them, respectively, in the following manner (with the addition of the formulas for calculating nonmilk products):

$$(1)\ \text{Pounds of serum-solids concentrate} = \frac{\left[\begin{array}{l}\text{pounds of milk-}\\ \text{solids-not-fat in}\\ \text{the mix}\end{array}\right] - \left[0.09 \times \text{pounds of mix serum}\right]}{\left[\begin{array}{l}\text{fraction of milk-}\\ \text{solids-not-fat in}\\ \text{the serum-solids}\\ \text{concentrate}\end{array}\right] - \left[0.09 \times \begin{array}{c}\text{fraction of serum}\\ \text{in the serum-solids}\\ \text{concentrate}\end{array}\right]}$$

$$(2)\ \text{Pounds of cream} = \frac{\left[\begin{array}{lll}\text{total} & & \text{pounds fat}\\ \text{pounds} & - & \text{in serum-}\\ \text{of fat} & & \text{solids}\\ \text{in mix} & & \text{concentrate}\end{array}\right] - \left[\begin{array}{lll}\text{fat} & & \text{pounds}\\ \text{fraction} & \times & \text{milk}\\ \text{of} & & \text{and}\\ \text{milk} & & \text{cream}\end{array}\right]}{\text{fraction of fat in cream} - \text{fraction of fat in milk}}$$

(3) Pounds of milk = pounds milk and cream minus pounds cream
(4) Pounds of sugar = pounds of sugar desired minus pounds of sugar added in serum-solids concentrate
(5) Pounds of gelatin = weight of mix × fraction of gelatin in mix

[4] Z can be condensed skim or whole milk, either sweetened or unsweetened; or whole or skim-milk powder.

Problem: Suppose 100 pounds of mix must contain 13.0 per cent fat, 10.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.4 per cent gelatin. Consider the following products available: cream containing 40.0 per cent fat; milk containing 4.0 per cent fat; sweetened condensed whole milk containing 8.0 per cent fat, 20.0 per cent milk-solids-not-fat, 42.0 per cent sugar;[5] and the nonmilk products, sugar and gelatin. The mix contains 84.6 pounds milk products of which 71.6 pounds is serum. By application of the formulas given above, the following results are obtained:

(1) Pounds of condensed milk $= \dfrac{10.00 - (0.09 \times 71.6)}{0.20 - (0.09 \times 0.50)} = 22.94$

(2) Pounds of cream $= \dfrac{13.00 - (0.08 \times 22.94) - (0.04 \times 71.29)}{0.40 - 0.04} = 23.09$

(3) Pounds of milk $= 71.29 - 23.09 = 48.20$

(4) Pounds of sugar $= 15.00 - (22.94 \times 0.42) = 5.37$

(5) Pounds of gelatin $= 100 \times 0.004 = 0.40$

Tabulated check 2 shows the figures to be correct.

TABULATED CHECK 2

Ingredients used	Weight	Milk fat supplied	Milk-solids-not-fat supplied	Sugar supplied	Total solids supplied
	pounds	*pounds*	*pounds*	*pounds*	*pounds*
Condensed whole milk testing 8.0 per cent fat, 20.0 per cent milk-solids-not-fat, 42.0 per cent sugar	22.94	1.84	4.59	9.63	16.06
Cream testing 40.0 per cent fat	23.09	9.24	1.25		10.49
Milk testing 4.0 per cent fat	48.20	1.93	4.16		6.09
Sugar	5.37			5.37	5.37
Gelatin	0.40				0.40
Totals supplied	100.00	13.01	10.00	15.00	38.41
Totals required	100.00	13.00	10.00	15.00	38.40

TABULATION AS A SHORT-CUT FOR ROUTINE CALCULATIONS

In many plants a mix of uniform composition is made from day to day; in addition the composition of the serum-solids concentrate used does not fluctuate. When such is the case, the amount of serum-solids concentrate required for a given-sized batch will be constant, as also will

[5] The fraction of serum in the serum-solids concentrate is calculated by subtracting from the whole, the fraction of fat and the fraction of added sugar. Thus, in this case 1.00 minus (0.08 fat plus 0.42 sugar) equals 0.50 serum. This equals the value of e in formula (1).

be the amounts of nonmilk products, no matter by which method they are calculated. A table posted in the work room giving the quantities of these products, along with the total weight and the average fat test of the milk and cream mixture required for the remainder of the standardization, will make unnecessary the repeated calculations for these values.

Table 1 illustrates such a tabulation for specified quantities of a mix to contain 12.0 per cent fat, 10.5 per cent milk-solids-not-fat, 15.0 per cent sugar and 0.35 per cent gelatin making 37.85 per cent total solids;

TABLE 1

INGREDIENTS REQUIRED FOR BATCHES OF ICE CREAM MIX CONTAINING 12.0 PER CENT FAT, 10.5 PER CENT MILK-SOLIDS-NOT-FAT, 15.0 PER CENT SUGAR, AND 0.35 PER CENT GELATIN

Mix	Standardized milk-and-cream mixture testing 17.8 per cent fat*	Condensed skim milk testing 32.0 per cent milk-solids-not-fat	Sugar	Gelatin
pounds	*pounds*	*pounds*	*pounds*	*pounds*
100	67.43	17.22	15.00	0.35
200	134.86	34.44	30.00	0.70
300	202.29	51.66	45.00	1.05
400	269.72	68.88	60.00	1.40
500	337.15	86.10	75.00	1.75
600	404.58	103.32	90.00	2.10
700	472.01	120.54	105.00	2.45
800	539.44	137.76	120.00	2.80
900	606.87	154.98	135.00	3.15

* A milk-and-cream mixture testing 17.8 per cent fat can easily be standardized by the Pearson square method.

where condensed skim milk testing 32.0 per cent milk-solids-not-fat is used as the serum-solids concentrate. Since the percentage of fat in milk and cream, respectively, can be expected to vary considerably from day to day, it usually is not practical to tabulate these for specified fat contents. The exact weight of each, however, can easily be calculated by the Pearson square method each time a mix is made. These are the only calculations required for routine standardization of such mixes. Thus, the final step in proportioning the ingredients for the problem illustrated in table 1, consists in calculating by this method the exact quantities of milk and cream with known fat contents respectively, to supply the weights of the mixture testing 17.80 per cent fat, which are indicated in column 2 of table 1 for various-sized batches.

The recipes given in table 2 likewise illustrate this method. They are designed to give quantities of ingredients required for 100-pound batches where different serum-solids concentrates are used; also where the composition of the mixes varies.

TABLE 2

TABULATED RECIPES FOR ICE MILK AND ICE CREAM MIXES

Desired composition of ice milk and ice cream mixes				Use any one of the following combinations to make 100 pounds of mix to give the desired indicated composition									
				Combination 1					Combination 2				
							Milk-and-cream mixture*					Milk-and-cream mixture*	
Fat	Milk-solids-not-fat	Sugar	Gelatin†	Sugar	Gelatin	Skim milk powder testing 97.0 per cent total solids	Weight	Fat test	Sugar	Gelatin	Condensed skim milk testing 32.0 per cent total solids	Weight	Fat test
per cent	*per cent*	*per cent*	*per cent*	*pounds*	*pounds*	*pounds*	*pounds*	*per cent*	*pounds*	*pounds*	*pounds*	*pounds*	*per cent*
4.50	13.00	15.00	0.50	15.00	0.50	6.59	77.91	5.78	15.00	0.50	25.22	59.28	7.59
4.50	15.00	15.00	0.50	15.00	0.50	8.86	75.64	5.95	15.00	0.50	33.91	50.59	8.90
6.00	12.50	15.00	0.50	15.00	0.50	6.18	78.32	7.66	15.00	0.50	23.60	60.90	9.85
6.00	14.00	15.00	0.50	15.00	0.50	7.88	76.62	7.83	15.00	0.50	30.15	54.35	11.04
10.50	10.50	15.00	0.40	15.00	0.40	4.35	80.25	13.08	15.00	0.40	16.65	67.95	15.45
10.50	11.50	15.00	0.40	15.00	0.40	5.49	79.11	13.27	15.00	0.40	21.00	63.60	16.51
12.00	9.50	15.00	0.40	15.00	0.40	3.38	81.22	14.77	15.00	0.40	12.91	71.69	16.74
12.00	10.50	15.00	0.40	15.00	0.40	4.51	80.09	14.98	15.00	0.40	17.26	67.34	17.82
14.00	9.00	15.00	0.30	15.00	0.30	3.00	81.70	17.14	15.00	0.30	11.48	73.22	19.12
14.00	10.00	15.00	0.30	15.00	0.30	4.14	80.56	17.38	15.00	0.30	15.83	68.87	20.33
16.00	8.00	15.00	0.30	15.00	0.30	2.07	82.63	19.36	15.00	0.30	7.91	76.79	20.84
16.00	9.00	15.00	0.30	15.00	0.30	3.20	81.50	19.63	15.00	0.30	12.26	72.44	22.09

TABLE 2—*(Concluded)*

Desired composition of ice milk and ice cream mixes				Use any one of the following combinations to make 100 pounds of mix to give the desired indicated composition									
				Combination 3					Combination 4				
							Milk-and-cream mixture*					Milk-and-cream mixture*	
Fat	Milk-solids-not-fat	Sugar	Gelatin†	Sugar	Gelatin	Evaporated milk testing 8.0 per cent fat and 20.0 per cent milk-solids-not-fat	Weight	Fat test	Sugar	Gelatin	Sweetened cond. skim milk testing 28.0 per cent milk-solids-not-fat and 42.0 per cent sucrose	Weight	Fat test
per cent	*per cent*	*per cent*	*per cent*	*pounds*	*pounds*	*pounds*	*pounds*	*per cent*	*pounds*	*pounds*	*pounds*	*pounds*	*per cent*
4.50	13.00	15.00	0.50	15.00	0.50	49.49	35.01	1.54	4.31	0.50	25.46	69.73	6.45
4.50	15.00	15.00	0.50	‡					0.62	0.50	34.24	64.64	6.96
6.00	12.50	15.00	0.50	15.00	0.50	46.35	38.15	6.01	4.98	0.50	23.85	70.67	8.49
6.00	14.00	15.00	0.50	15.00	0.50	59.10	25.40	5.00	2.22	0.50	30.42	66.86	8.98
10.50	10.50	15.00	0.40	15.00	0.40	32.68	51.92	15.20	7.94	0.40	16.81	74.85	14.03
10.50	11.50	15.00	0.40	15.00	0.40	41.20	43.40	16.60	6.10	0.40	21.20	72.30	14.52
12.00	9.50	15.00	0.40	15.00	0.40	25.34	59.26	16.82	9.52	0.40	13.04	77.04	15.58
12.00	10.50	15.00	0.40	15.00	0.40	33.87	50.73	18.31	7.68	0.40	17.43	74.49	16.11
14.00	9.00	15.00	0.30	15.00	0.30	22.53	62.17	19.62	10.13	0.30	11.59	77.98	17.95
14.00	10.00	15.00	0.30	15.00	0.30	31.06	53.64	21.48	8.29	0.30	15.98	75.43	18.56
16.00	8.00	15.00	0.30	15.00	0.30	15.53	69.17	21.34	11.64	0.30	7.99	80.07	19.98
16.00	9.00	15.00	0.30	15.00	0.30	24.06	60.64	23.22	9.80	0.30	12.38	77.52	20.64

* The desired composition of the milk-and-cream mixture in each case can be secured by using the Pearson square method of standardization, described elsewhere in this publication.

† The percentage of gelatin used in any one case should depend upon the composition of the mix and the grade of gelatin available. The required gelatin content, with a given grade of gelatin, will decrease with an increase in the fat or total-solids content of the mix.

‡ It is not possible to obtain the desired composition if evaporated milk is used in this case.

ALGEBRAIC METHOD

The algebraic method here illustrated depends upon the use of simultaneous equations. The most common type of mix standardization problem involves the use of three unknowns and three equations, as illustrated below.

Suppose it is desired to make 100 pounds of mix to contain 14.0 per cent fat, 9.0 per cent milk-solids-not-fat, 14.5 per cent sugar, 0.5 per

TABLE 3

RELATION OF PERCENTAGE OF FAT TO PERCENTAGE OF OTHER SOLIDS IN MILK*

Milk fat	Milk-solids-not-fat	Total milk solids	Ratio of fat to solids-not-fat	Calculated milk-solids-not-fat in skim-milk portion (serum)
per cent	*per cent*	*per cent*		*per cent*
3.0	8.33	11.33	1 : 2.77	8.59
3.1	8.40	11.50	1 : 2.71	8.67
3.2	8.46	11.66	1 : 2.64	8.74
3.3	8.52	11.82	1 : 2.58	8.81
3.4	8.55	11.95	1 : 2.52	8.85
3.5	8.60	12.10	1 : 2.46	8.91
3.6	8.65	12.25	1 : 2.40	8.97
3.7	8.69	12.39	1 : 2.35	9.02
3.8	8.72	12.52	1 : 2.30	9.06
3.9	8.76	12.66	1 : 2.25	9.12
4.0	8.79	12.79	1 : 2.20	9.16
4.1	8.82	12.92	1 : 2.15	9.20
4.2	8.86	13.06	1 : 2.11	9.25
4.3	8.89	13.19	1 : 2.07	9.29
4.4	8.92	13.32	1 : 2.03	9.33
4.5	8.95	13.45	1 : 1.99	9.37
4.6	8.98	13.58	1 : 1.95	9.41
4.7	9.01	13.71	1 : 1.92	9.45
4.8	9.04	13.84	1 : 1.88	9.50
4.9	9.07	13.97	1 : 1.85	9.54
5.0	9.10	14.10	1 : 1.82	9.58

* The first four columns of this table are taken from: Kelley, E., and C. E. Clements. Market milk, 2nd ed., p. 42. John Wiley and Sons, New York, 1931. The values given in this table represent averages compiled from more than 250,000 milk samples.

cent gelatin, and 0.5 per cent egg-yolk solids. Consider the following ingredients available: cream testing 30.0 per cent fat; milk testing 3.6 per cent fat; evaporated whole milk containing 8.0 per cent fat and 20.0 per cent milk-solids-not-fat; egg yolk containing 50.0 per cent egg-yolk solids; sugar; and gelatin.

In some cases analyses of milk and cream for both fat and total solids are made, but usually only the percentage of fat is determined. Tables

3 and 4 show the relations between fat and serum solids that are likely to occur, and the values for milk-solids-not-fat given can be used where

TABLE 4

Percentages of Milk-Solids-Not-Fat in Cream Where Its Serum Contains the Percentages of Milk-Solids-Not-Fat Given in the Headings of the Respective Columns

Fat in cream	Milk-solids-not-fat in cream where the serum contains					
	8.7	8.8	8.9	9.0*	9.1	9.2
15.0	7.40	7.48	7.57	7.65	7.74	7.82
16.0	7.31	7.39	7.48	7.56	7.64	7.73
17.0	7.22	7.30	7.39	7.47	7.55	7.64
18.0	7.13	7.21	7.30	7.38	7.46	7.54
19.0	7.05	7.13	7.21	7.29	7.37	7.45
20.0	6.96	7.04	7.12	7.20	7.28	7.36
21.0	6.87	6.95	7.03	7.11	7 19	7.27
22 0	6.79	6.86	6.94	7.02	7.10	7 18
23.0	6.70	6.78	6.85	6.93	7.01	7.08
24 0	6.61	6.69	6.76	6 84	6.92	6.99
25.0	6.53	6.60	6.68	6.75	6.83	6.90
26.0	6.44	6 51	6.59	6.66	6.73	6.81
27.0	6.35	6.42	6.50	6.57	6.64	6.72
28 0	6.26	6.34	6.41	6.48	6.55	6.62
29.0	6.18	6.25	6.32	6.39	6.46	6.53
30.0	6.09	6.16	6.23	6.30	6.37	6.44
31.0	6.00	6.07	6.14	6.21	6.28	6.35
32.0	5.92	5.98	6.05	6 12	6.19	6.26
33.0	5.83	5.90	5.96	6.03	6.10	6.16
34.0	5.74	5.81	5.87	5.94	6.01	6.07
35.0	5.66	5.72	5.79	5.85	5 92	5.98
36.0	5.57	5 63	5.70	5.76	5.82	5.89
37.0	5.48	5.54	5.61	5.67	5.73	5.80
38.0	5.39	5.46	5.52	5.58	5.64	5.70
39.0	5.31	5.37	5.43	5.49	5.55	5 61
40.0	5.22	5.28	5.34	5.40	5.46	5 52
41.0	5.13	5.19	5.25	5.31	5.37	5.43
42.0	5.05	5 10	5.16	5.22	5.28	5.34
43.0	4.96	5.02	5.07	5.13	5.19	5.24
44.0	4 87	4.93	4 98	5.04	5.10	5.15
45.0	4.79	4.84	4.90	4.95	5.01	5.06
46.0	4 70	4.75	4.81	4.86	4.91	4.97
47.0	4.61	4.66	4 72	4.77	4.82	4.88
48.0	4.52	4.58	4.63	4.68	4 73	4.78
49.0	4.44	4.49	4.54	4.59	4.64	4.69
50 0	4.35	4.40	4 45	4.50	4.55	4.60
55.0	3.92	3.96	4.01	4 05	4 10	4 14
60.0	3.48	3.52	3.56	3.60	3.64	3.68

* A safe average for practical purposes is 9.0 per cent milk-solids-not-fat in the serum of milk and cream. Hence, the values in this column should be used unless it is known that some other value more nearly represents the composition of the sample under consideration.

analytical data are lacking. If in any case it is known that some other values more nearly represent the compositions of the products, obviously they should be used.

The amounts of nonmilk products are found as follows:

$$0.145 \times 100 \text{ pounds} = 14.5 \text{ pounds sugar}$$
$$0.005 \times 100 \text{ pounds} = 0.5 \text{ pounds gelatin}$$
$$\frac{0.005 \times 100 \text{ pounds}}{0.5} = 1.0 \text{ pounds egg yolk (contains 0.5 pound egg yolk solids)}$$

Hence, $(14.5 + 0.5 + 1.0)$ pounds $= 16.0$ pounds of nonmilk products; and $(100.0 - 16.0)$ pounds $= 84.0$ pounds of milk products.

In order to solve for the amount of each of the milk products, they will be designated as follows:

X = pounds of 30.0 per cent cream required
Y = pounds of 3.6 per cent milk required
Z = pounds of evaporated milk required

Three equations[6] using these three unknowns follow:

(a) Equation for total milk products: $X + Y + Z = 84.0$ pounds
(b) Equation for fat in mix: $0.30X + 0.036Y + 0.08Z = 14.0$ pounds
(c) Equation for serum solids in mix: $0.063X + 0.0865Y + 0.20Z = 9.0$ pounds

These equations can be solved by eliminating one or more unknowns at a time; either by combining two equations by addition or subtraction, or by substituting the value for one unknown in another equation. These are commonly called the addition or subtraction method, and the substitution method, respectively. Usually with three unknowns the first method is preferable, although sometimes a combination of the two methods makes the calculations easier. Only the first method will be illustrated here. In this case, one of the equations will have to be multiplied by a factor such that the coefficients of one of the unknowns in two equations are the same. Thus, if equation (c) above is multiplied by 5, the coefficient of Z becomes the same as the coefficient of Z in equation (a). Subtracting,

(a)	$X + Y + Z = 84.00$	
(d)	$0.315X + 0.4325Y + Z = 45.00$	(Equation $c \times 5$)
(e)	$0.685X + 0.5675Y = 39.00$	(Equation a — equation d)

Similarly, Z can be eliminated from equations (a) and (b) if equation (a) is multiplied by 8, and equation (b) by 100. Subtracting,

(f)	$30X + 3.6Y + 8Z = 1{,}400.00$	(Equation $b \times 100$)
(g)	$8X + 8.0Y + 8Z = 672.00$	(Equation $a \times 8$)
(h)	$22X - 4.4Y = 728.00$	(Equation f — equation g)

[6] If the condensed milk used contains sugar, proper allowance should be made for this in the equations. Thus if it contains 42.0 per cent sucrose, equation (a) above would become $X + Y + 0.58Z = 84.0$.

One more unknown can be eliminated from equations (e) and (h) by dividing each equation by its respective coefficient of X. Subtracting,

(i) $X + 0.8285Y = 56.934$ (Equation $e \div 0.685$)
(j) $X - 0.2000Y = 33.091$ (Equation $h \div 22.0$)
(k) $1.0285Y = 23.843$ (Equation i — equation j)

Solving equation (k) for Y:

$$Y = 23.182$$

Substitute this value of Y in equation (h) and solve for X:

$$22X - 4.4 \times 23.182 = 728.000$$
$$22X = 830.001$$
$$X = 37.727$$

Substitute values for X and Y in equation (a) and solve for Z:

$$37.727 + 23.182 + Z = 84.0$$
$$Z = 23.091$$

The results are verified as shown in tabulated check 3.

TABULATED CHECK 3

Ingredients	Weight	Milk fat supplied	Milk-solids-not-fat supplied	Total solids supplied
	pounds	*pounds*	*pounds*	*pounds*
Sugar	14.50			14.50
Gelatin	0.50			0.50
Egg yolk	1.00			0 50
Milk testing 3.6 per cent fat	23.182	0.835	2.005	2.84
Evaporated milk testing 8.0 per cent fat, 20.0 per cent milk-solids-not-fat	23.091	1.847	4.618	6.465
Cream testing 30.0 per cent fat	37.727	11.318	2.377	13.695
Totals supplied	100.000	14.000	9.000	38.500
Totals required	100.000	14.000	9.000	38.500

GRAPHIC METHOD OF STANDARDIZATION

The graphic method of standardizing ice cream mixes, introduced by J. A. Cross[7] and later reported by W. V. Price[8] as applicable for re-standardization as well as for original standardization problems, should prove satisfactory for those accustomed to the use of graphs. The method is simple, and the calculations required are relatively few. Inaccuracies

[7] Mojonnier, T., and H. C. Troy. Technical control of dairy products. Mojonnier Brothers Company, Chicago. p. 416–418. 1922.

[8] Price, W. V. A graphical method of proportioning and standardizing ice cream mixes. Journal Dairy Science 10:292–299. 1927.

introduced by plotting and reading of the graph need not be great enough to make the method impracticable for general use. In fact, its simplicity and fair degree of accuracy and especially its adaptability for restandardizing off-batches recommend its more general use.

The method can perhaps best be explained by an illustration. Suppose it is desired to make 100 pounds of mix to test 13.0 per cent fat, 10.5 per cent milk-solids-not-fat, 14.6 per cent sugar, and 0.4 per cent gelatin; and the products available are sugar, gelatin, cream testing 33.0 per cent fat, milk testing 3.6 per cent fat, and evaporated milk testing 8.0 per cent fat and 20.0 per cent milk-solids-not-fat.

The calculations for the nonmilk products are the same as in any of the other methods previously considered. Thus,

$$0.146 \times 100 \text{ pounds} = 14.6 \text{ pounds sugar; and,}$$
$$0.004 \times 100 \text{ pounds} = 0.4 \text{ pound gelatin.}$$

$$\text{Hence, } (14.6 + 0.4) \text{ pounds} = 15.0 \text{ pounds of nonmilk products; and,}$$
$$(100.0 - 15.0) \text{ pounds} = 85.0 \text{ pounds of milk products.}$$

In order to solve the problem by this method, first determine the percentage of fat and the percentage of milk-solids-not-fat in the milk products. Thus,

$$\frac{13.0}{85} \times 100 = 15.29 \text{ per cent fat; and}$$

$$\frac{10.5}{85} \times 100 = 12.35 \text{ per cent milk-solids-not-fat.}$$

Using coördinate paper as indicated in figure 1, let the horizontal axis represent the percentage of milk-solids-not-fat, and let the vertical axis represent the percentage of fat. Now, on this graph, plot point *A*, using the values just calculated to represent the composition of the milk products portion of the mix to be standardized. In addition to using the data given in the problem, refer to tables 3 and 4 for the percentages of milk-solids-not-fat in the milk and cream; then using these values, plot points representing the composition of the three milk products available for standardization: *B* representing the evaporated milk; *C* the milk; and *D* the cream.

Obviously, the line segment *CD* represents the composition of all possible mixtures of milk and cream represented by the points *C* and *D*. Now the line *BA*, when extended, will intersect the line *CD* at *E*, representing the composition of the milk-and-cream mixture, which when combined in the correct proportion with the evaporated milk of *B* composition, will result in a mixture of *A* composition.[9] The other desig-

[9] When used for routine standardization the work can be simplified if a large sheet of coördinate paper is used and the necessary points represented by pins. Lines can be indicated by thread stretched between the pins.

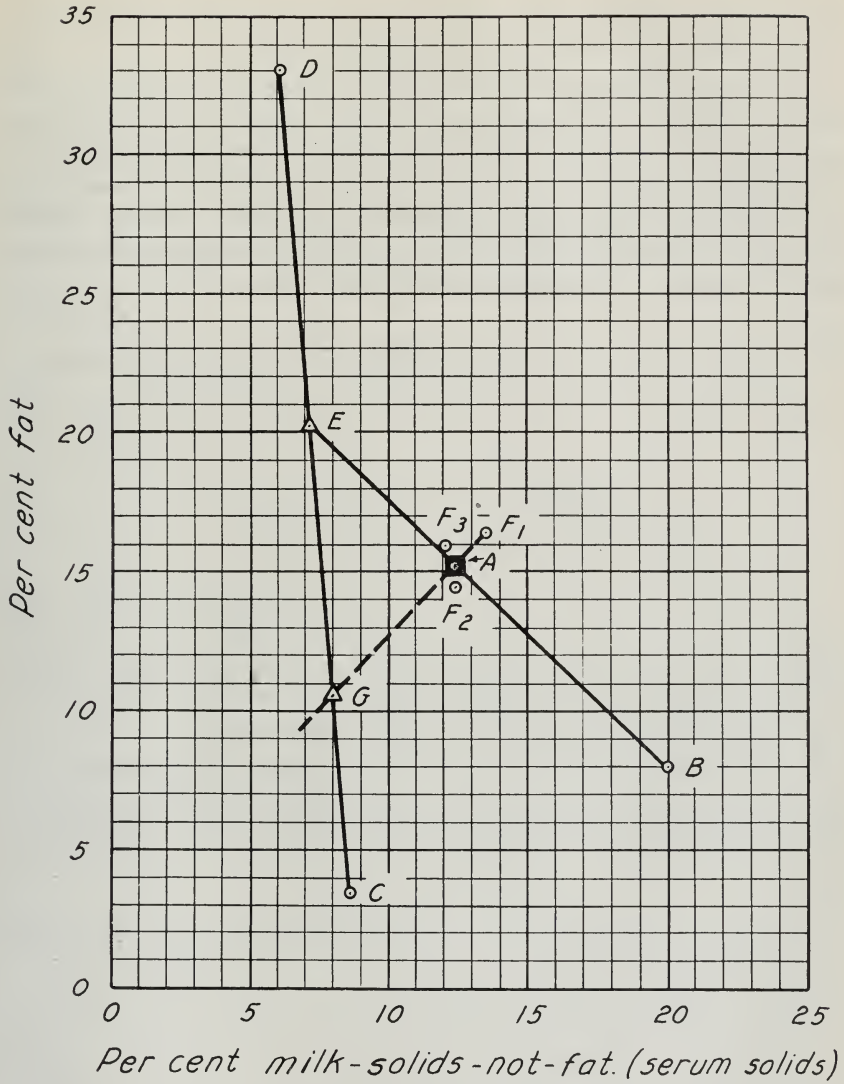


Fig. 1.—Graph for standardizing mixes and restandardizing off-batches.

A, Composition of dairy products portion of desired mix.
B, Composition of condensed milk available.
C, Composition of milk available.
D, Composition of cream available.
E, and *G*, Composition of two milk-and-cream mixtures determined graphically.
F_1, F_2, and F_3, Compositions of the dairy products portions of three off-batches.

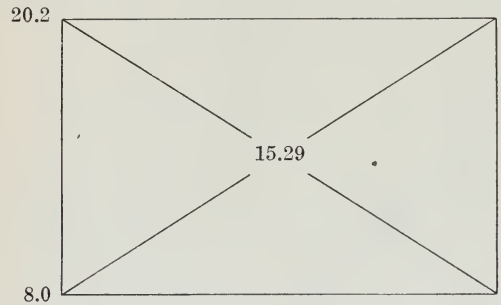
nated points in figure 1 can be ignored for the present but will be used later in a problem of restandardization presented on pages 27 and 28.

The problem can be finished in either of two ways:

Procedure 1: Using the Pearson square, standardize in each case for either the milk fat or the milk-solids-not-fat. Usually it is preferable to standardize on the basis of the fat as illustrated in the problem below.

Determine the pounds of evaporated milk by taking into consideration the fact that evaporated milk of *B* composition contains 8.0 per cent fat and the milk-and-cream mixture of *E* composition contains 20.2 per cent fat, as can be seen by referring to the graph.

Thus,



7.29 = unit parts of milk and cream mixture (*E*)

4.91 = unit parts of evaporated milk (*B*)

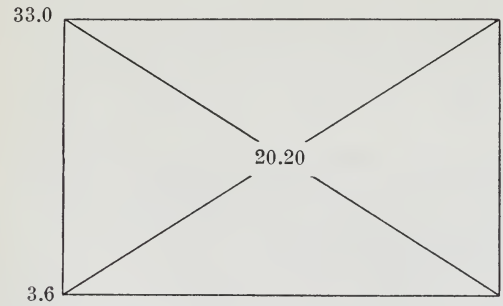
Hence, (7.29 + 4.91) = 12.20 = total unit parts of mixture containing 15.29 per cent fat; and, (*A*)

$$\text{the pounds of evaporated milk} = \frac{4.91}{12.20} \times 85.0 = 34.21.$$

Therefore, the pounds of milk-and-cream mixture = (85.00 — 34.21) = 50.79.

Next determine the pounds of milk and cream required to make 50.79 pounds of the mixture containing 20.20 per cent fat.

Thus,



16.60 = unit parts of cream (*D*)

12.80 = unit parts of milk (*C*)

Hence, (16.60 + 12.80) = 29.40 = total unit parts of milk-and-cream mixture; and, (*E*)

$$\text{the pounds of cream} = \frac{16.60}{29.40} \times 50.79 = 28.68.$$

Therefore, the pounds of milk = 50.79 — 28.68 = 22.11.

Procedure 2: The lengths of line segments on the graph can be used to determine the proportion of the ingredients required. The unit of length chosen is immaterial as long as it is uniform throughout the calculations, but usually the most convenient for this purpose is the unit of space on the graph. The length in any case can be obtained by using dividers and reading the length off the base line or by simply subtracting on one of the scales the difference between two points; for example, length of line segment $CD = (33.00 - 3.60) = 29.40$; length of line segment $AB = (15.29 - 8.00) = 7.29$. By this last method of determining the length, the pounds of each ingredient can be calculated as follows:

$$\text{Pounds of evaporated milk} = \frac{\text{length of line } EA}{\text{length of line } EB} \times \text{pounds of milk products};$$

$$\frac{4.91}{12.20} \times 85.0 = 34.21$$

$$\text{Pounds of milk-and-cream mixture} = (85.00 - 34.21) = 50.79$$

$$\text{Pounds of cream} = \frac{\text{length of line } CE}{\text{length of line } CD} \times \text{pounds of milk and cream};$$

$$\frac{16.60}{29.40} \times 50.79 = 28.68$$

$$\text{Pounds of milk} = (50.79 - 28.68) = 22.11$$

When this method of determining the length of the line segment is used, the calculations are identical with those by the Pearson square method, except that ordinarily the measurements cannot be scaled so accurately as they can be computed.

Tabulated check 4 shows that the calculations are correct for practical purposes.

TABULATED CHECK 4

Ingredients	Weight	Milk fat	Milk-solids-not-fat	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Sugar	14.60			14.60
Gelatin	0.40			0.40
Evaporated milk testing 8.0 per cent fat and 20.0 per cent milk-solids-not-fat	34.21	2.74	6.84	9.58
Cream testing 33.0 per cent fat	28.68	9.46	1.73	11 19
Milk testing 3.6 per cent fat	22.11	0.80	1.91	2.71
Totals supplied	100.00	13.00	10.48	38 48
Totals required	100 00	13 00	10.50	38 50

STANDARDIZING MIXES TO BE MADE IN A VACUUM PAN

The calculations for standardizing mixes to be prepared in a vacuum pan are fewer than in the more usual process where a serum-solids concentrate is used. The weights of nonmilk products are calculated for a definite finished weight of mix. Then the composition of a mixture of milk and cream having the correct ratio of $\frac{\text{fat}}{\text{milk-solids-not-fat}}$ is calculated, or determined by reference to a table such as table 5 or a graph such as figure 2; and finally the weight of this mixture required is calculated by the Pearson square method. The following example will illustrate this procedure.

Problem: Determine the amounts of ingredients required to prepare 1,000 pounds of mix, the latter to be made in a vacuum pan. The desired composition of the mix is 14.0 per cent fat, 10.0 per cent milk-solids-not-fat, 14.7 per cent sugar, and 0.3 per cent gelatin. Milk testing 3.7 per cent fat and cream testing 40.0 per cent fat are available in addition to sugar and gelatin.

The order in which the ingredients are determined is of no consequence in this case, but for convenience the calculations are listed in steps.

Calculate the amounts of nonmilk products required:

$0.147 \times 1{,}000$ pounds $= 147.0$ pounds sugar; and
$0.003 \times 1{,}000$ pounds $= 3.0$ pounds gelatin.

Calculate the ratio of $\frac{\text{fat}}{\text{milk-solids-not-fat}}$ desired in the final mix. Thus,

$$\frac{14.0}{10.0} = 1.40.$$

Determine the percentage of fat in the mixture of milk and cream having the ratio 1.40 by referring to either table 5 or figure 2. This is found to be 11.2 per cent fat.

It may also be calculated in the following manner:

Let F = fat fraction of the desired mix
S = milk-solids-not-fat fraction of the desired mix
A = milk-solids-not-fat fraction of the serum (skim-milk portion) of the mixture of milk and cream
X = fat fraction of the desired milk-and-cream mixture

Hence, $(1 - X)A$ = milk-solids-not-fat fraction of the milk-and-cream mixture; and

$$\frac{F}{S} = \frac{X}{(1-X)A}$$

Solving this equation for X,

$$X = \frac{AF}{S + AF}$$

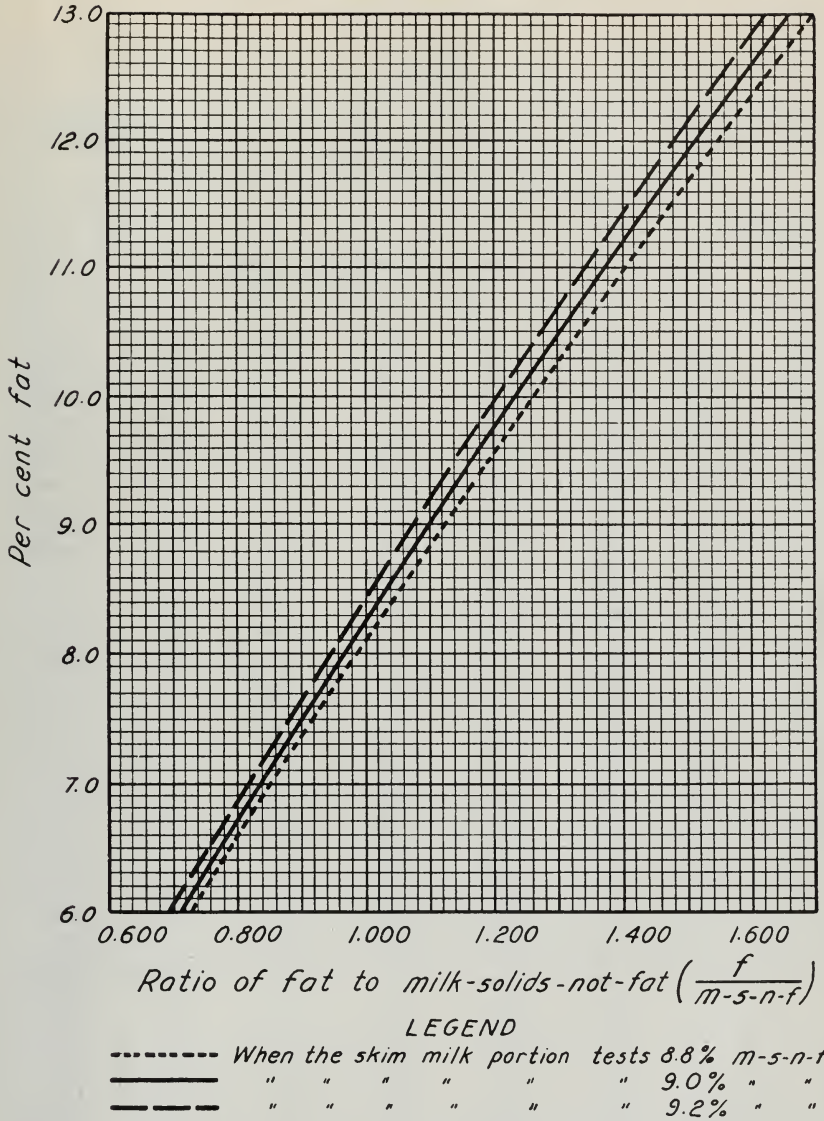


Fig. 2.—Graph showing the percentage of fat in milk-and-cream mixtures as related to the ratio of fat to milk solids-not-fat $\left[\frac{\text{fat}}{\text{milk-solids-not-fat}}\right]$ in such mixtures.

Assuming that the serum tests 9.0 per cent milk-solids-not-fat, the value of A becomes 0.09. When substituted in the preceding equation, it is as follows:

$$X = \frac{0.09F}{S + 0.09F}$$

TABLE 5

RATIOS OF FAT TO MILK-SOLIDS-NOT-FAT IN MIXTURES OF MILK AND CREAM

Per cent fat in mixture	Per cent milk-solids-not-fat in *serum* of mixture					
	8.7	8.8	8.9	9.0†	9.1	9.2
	Ratio of fat to milk-solids-not-fat in *mixture* (milk-solids-not-fat = 1)					
6.0	0.733	0.726	0.717	0.709	0.702	0.694
6.1	0.747	0.738	0.730	0.722	0.714	0.706
6.2	0.760	0.752	0.743	0.735	0.726	0.718
6.3	0.773	0.765	0.755	0.747	0.739	0.731
6.4	0.786	0.777	0.768	0.760	0.751	0.743
6.5	0.800	0.790	0.781	0.772	0.764	0.756
6.6	0.812	0.803	0.794	0.785	0.776	0.768
6.7	0.825	0.816	0.807	0.798	0.789	0.781
6.8	0.838	0.829	0.820	0.810	0.802	0 793
6.9	0.852	0.842	0.832	0.823	0.815	0 805
7.0	0 865	0.856	0.845	0.836	0.827	0.818
7.1	0.879	0.868	0.859	0.849	0.840	0.830
7.2	0.892	0.881	0.872	0.862	0.853	0.843
7.3	0.906	0.895	0.885	0.875	0.865	0.856
7.4	0.918	0.908	0.898	0.888	0.878	0.869
7.5	0.932	0.921	0.911	0.900	0.891	0.881
7.6	0.945	0.935	0.925	0.913	0.904	0.894
7.7	0.959	0.948	0.938	0.927	0.917	0.907
7.8	0.973	0.962	0.950	0.940	0.930	0.920
7.9	0.986	0.975	0.963	0.953	0 943	0.933
8.0	1.000	0.988	0.977	0.966	0.956	0.946
8.1	1.013	1.001	0.990	0.979	0.969	0.959
8.2	1.026	1.015	1.004	0.993	0.982	0 970
8.3	1.040	1.029	1.017	1.006	0.995	0.983
8.4	1.054	1.042	1.031	1.019	1.007	0.996
8.5	1.068	1.056	1.044	1.032	1.020	1.010
8.6	1.082	1.070	1.058	1.045	1.034	1.023
8.7	1.096	1.083	1.070	1.058	1.047	1.036
8.8	1.110	1.096	1.084	1.072	1.060	1.049
8.9	1.122	1.110	1.097	1.085	1.074	1.062
9 0	1.136	1.124	1.111	1.099	1.087	1.075
9.1	1.150	1.138	1.125	1.112	1.100	1.089
9.2	1.165	1.151	1.139	1.126	1.114	1.102
9.3	1.179	1.165	1.152	1.140	1.127	1.115
9.4	1.193	1.179	1.166	1.153	1.141	1.127
9.5	1.207	1.193	1.180	1.166	1.153	1.140
9.6	1.221	1.206	1.193	1.179	1.166	1.154
9.7	1.234	1.220	1.206	1.193	1.180	1.167
9.8	1.248	1.234	1 220	1.207	1.194	1.181
9.9	1.263	1.248	1.234	1.221	1.207	1.194

* The ratios in this table were derived by dividing the percentage of fat by the percentage of milk-solids-not-fat for each mixture. Thus the first value in the table is obtained as follows: A mixture containing 6 per cent fat will contain 94 per cent serum. As indicated in the column heading, the serum contains 8.7 per cent milk-solids-not-fat. Hence, the mixture contains (0.94×8.7) or 8.178 per cent milk-solids-not-fat and $\frac{6.0}{8.178} = 0.733$.

† Values given in this column should be used unless it is known for a specific case that a value given in one of the other columns more nearly represents the relation which exists between the fat and milk-solids-not-fat.

TABLE 5—*(Concluded)*

Per cent fat in mixture	Per cent milk-solids-not-fat in *serum* of mixture					
	8.7	8.8	8.9	9.0†	9.1	9.2
	Ratio of fat to milk-solids-not-fat in *mixture* (milk-solids-not-fat=1)					
10.0	1.277	1.263	1.248	1.235	1.221	1.208
10.1	1.292	1.277	1.263	1.248	1.235	1.221
10.2	1.306	1.291	1.277	1.262	1.248	1.235
10.3	1.321	1.305	1.291	1.276	1.262	1.248
10.4	1.333	1.320	1.305	1.290	1.276	1.262
10.5	1.348	1.332	1.317	1.303	1.290	1.276
10.6	1.362	1.347	1.332	1.317	1.302	1.290
10.7	1.377	1.361	1.346	1.331	1.316	1.302
10.8	1.392	1.376	1.360	1.345	1.330	1.315
10.9	1.406	1.390	1.375	1.359	1.344	1.329
11.0	1.421	1 405	1.389	1.373	1.358	1.343
11.1	1.436	1.419	1.403	1.388	1.372	1 357
11.2	1.449	1.434	1.418	1 402	1.386	1.371
11.3	1.464	1.447	1.432	1.416	1.400	1.385
11.4	1.479	1 462	1.445	1.430	1.414	1.399
11.5	1.494	1.476	1 459	1.443	1.429	1.413
11.6	1.508	1.491	1.474	1.457	1.443	1.427
11.7	1.523	1.506	1.489	1.472	1.455	1.441
11.8	1.538	1.521	1.503	1.486	1.469	1.455
11.9	1 554	1.535	1.518	1.501	1.484	1.467
12.0	1.567	1.550	1.533	1.515	1.498	1.481
12.1	1.582	1.563	1.547	1.530	1.513	1.496
12.2	1.597	1.578	1.562	1.544	1 527	1.510
12.3	1.612	1.593	1.575	1.559	1.541	1.524
12.4	1.627	1.608	1.590	1.574	1.556	1.538
12.5	1.643	1.623	1.605	1.586	1.570	1.553
12.6	1.658	1.638	1.620	1.601	1.585	1.567
12.7	1.671	1.654	1.634	1.616	1.599	1.582
12.8	1.686	1.669	1 649	1.631	1.612	1.596
12.9	1.702	1.684	1.665	1.645	1.627	1.610
13.0	1.717	1.697	1.680	1.660	1.641	1.625

† Values given in this column should be used unless it is known for a specific case that a value given in one of the other columns more nearly represents the relation which exists between the fat and milk-solids-not-fat.

In this problem $F = 0.14$ and $S = 0.10$. Hence,

$$X = \frac{0.09 \times 0.14}{0.10 + 0.09 \times 0.14} = 0.1119$$

This equals 11.19 per cent, or, for practical purposes, 11.2 per cent.

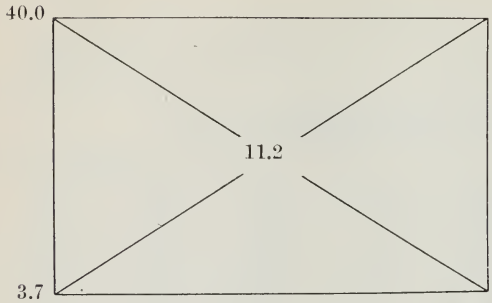
Then calculating the pounds of this milk-and-cream mixture required for the finished mix:

$0.14 \times 1{,}000 = 140.0$ pounds fat required, or

$\frac{140.00}{0.112} = 1{,}250$ pounds of mixture testing 11.2 per cent fat required.

Finally the pounds of milk and cream, respectively, required to prepare the above mixture are calculated by the Pearson square method.

Thus,



(7.5 + 28.8) = 36.3 = the total unit parts of the mixture testing 11.2 per cent fat.

Hence, the pounds of cream testing 40.0 per cent fat $= \frac{7.5}{36.3} \times 1{,}250 = 258.3$,

and the pounds of milk testing 3.7 per cent fat = (1,250.0 — 258.3) = 991.7.

Verification of these calculations will be found in tabulated check 5.

TABULATED CHECK 5

Ingredients	Weight	Milk fat	Milk-solids-not-fat	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Sugar	147.0			147.00
Gelatin	3.0			3.00
Cream testing 40.0 per cent fat	258.3	103.32	13.95	117.27
Milk testing 3.7 per cent fat	991.7	36.69	86.18	122.87
Totals supplied	1,400.0	140.01	100.13	390.14
Totals required	1,000.0	140.00	100.00	390 00

As shown in tabulated check 5 the required solids for 1,000 pounds of mix of the desired composition are supplied by a weight of 1,400 pounds. Hence, all that remains to be done is to evaporate the amount of water represented by the difference between that desired and the mixture above—that is, 1,400 — 1,000 = 400 pounds of water to be evaporated. The correct composition is determined by using a hydrometer that gives the specific gravity corresponding to a given total-solids content at the temperature of operation.

CORRECTING OFF-BATCHES BY RESTANDARDIZATION

Often an ice cream mix does not have the desired composition. If it deviates appreciably from the standard adopted the mix should be "restandardized"—that is, its composition should be adjusted to the desired values. Slight variations from the desired composition ordinarily need not be corrected.

The composition of the mix is usually determined by analyzing it for fat and total solids, then assuming that the nonmilk products are present in the correct proportion and computing the milk-solids-not-fat by difference. To illustrate this point, suppose a mix was found by analysis to contain 13.0 per cent fat and 39.5 per cent total solids, whereas it would have contained 12.5 per cent fat, 10.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.5 per cent gelatin, provided no mistake had occurred in the original standardization or processing. The sugar and gelatin contents are assumed to be correct, since, generally speaking, errors are less likely to occur in the sugar and gelatin contents of the mix than they are elsewhere. Hence the percentages of fat in the off-batch and the desired percentages of sugar and gelatin will all be subtracted from the total solids in the off-batch, as a basis of calculating the milk-solids-not-fat contained in the batch under consideration. Thus,

$$39.5 - (13.0 + 15.0 + 0.5) = 11.0 = \text{per cent of milk-solids-not-fat in the off-batch.}$$

Mixes can be restandardized by adding the minimum weight of ingredients to correct the off-batch or by simply adding a convenient weight of ingredients, provided the latter is large enough to permit the desired correction in composition to be made. In some cases there is a decided advantage in a minimum-sized final batch; in others, there is little or no choice between the two methods of deciding the weight of the final mix, except for convenience in calculating.

RESTANDARDIZING ON ARBITRARY-WEIGHT BASIS

In case the mix is restandardized by the use of an arbitrary quantity equaling or exceeding the minimum required, almost any method used for the original standardization can be employed. In such a case the off-batch is considered one of the ingredients that must be completely utilized. The remainder of the mix can be adjusted, and calculations for its standardization made, so that when the two portions are combined the final mixture will have the desired composition.

The following problem will illustrate restandardization where the mix is increased by an arbitrary quantity.

Problem: Restandardize 100 pounds of mix testing 12.5 per cent fat and 39.0 per cent total solids so that the final mixture will contain 14.0 per cent fat, 10.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.5 per cent gelatin. The milk products available are cream testing 30.0 per cent fat, milk testing 3.6 per cent fat, and condensed skim milk testing 32.0 per cent milk-solids-not-fat.

Arbitrarily assume that a total of 30 pounds will be added to correct the off-batch.

Tabulated check 6 summarizes the problem.

TABULATED CHECK 6

	Fat	Milk-solids-not-fat	Sugar	Gelatin	Total solids
	pounds	*pounds*	*pounds*	*pounds*	*pounds*
Amount required for 130 pounds corrected mix	18.20	13.00	19.50	0.65	51.35
Amount supplied by 100 pounds of the off-batch	12.50	11.00	15.00	0.50	39.00
Difference to be supplied in 30 pounds added for restandardization	5.70	2.00	4.50	0.15	12.35

Any one of the first four methods already illustrated in the section devoted to standardization can be used in solving this problem. Only the solution by the serum point method will be given here.

Tabulated check 6 shows that the correcting batch of 30.0 pounds must contain 4.65 pounds of nonmilk products and 25.35 pounds of milk products. The fat present in these added milk products amounts to 5.70 pounds; consequently, they will contain 19.65 pounds of serum which is the difference between the weight of milk products and the fat contained therein.

Proceed by substituting the correct values in the serum point formulas given on page 7:

$$\text{Pounds condensed skim milk required} = \frac{2.00 - 0.09 \times 19.65}{0.32 - 0.09} = 1.01$$

$$\text{Pounds milk and cream} = 25.35 - 1.01 = 24.34$$

$$\text{Pounds cream} = \frac{5.70 - 0.036 \times 24.34}{0.30 - 0.036} = 18.27$$

$$\text{Pounds milk} = 24.34 - 18.27 = 6.07$$

Tabulated check 7 verifies these calculations.

Where this method is employed, the procedure is the same whether the milk fat or milk-solids-not-fat in the off-batch are both too high, or both too low, or whether one is too high and the other too low. The only prerequisite is that the correcting batch equal or exceed the minimum required in any case.

TABULATED CHECK 7

Ingredients	Weight	Fat	Milk-solids-not-fat	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Off-batch	100.00	12.50	11.00	39.00
Sugar	4.50			4.50
Gelatin	0.15			0.15
Condensed skim milk testing 32.0 per cent serum solids	1.01		0.32	0.32
Cream testing 30.0 per cent fat	18.27	5.48	1.15	6.63
Milk testing 3.6 per cent fat	6.07	0.22	0.53	0.75
Totals supplied	130.00	18.20	13.00	51.35
Totals required	130.00	18.20	13.00	51.35

RESTANDARDIZING ON MINIMUM-WEIGHT BASIS

Ordinarily, though not always, it is an advantage to correct an off-batch by increasing the weight as little as possible. Whether or not this makes the problem more difficult will depend upon the correction to be made and upon the method employed for restandardization.

The graphic method permits the worker to restandardize on the basis of a minimum weight more easily than on the basis of any larger quantity, and to determine readily the ingredients required.

The algebraic method ordinarily involves no additional calculations to determine the minimum weight of the correcting batch, although in some cases it is not readily apparent which ingredients will be required for restandardization.

Formulas derived algebraically can likewise be used. A few of these are given later, and the methods of solution by their use illustrated.

Graphic Method of Restandardization.—In order to illustrate this method, reference will be made to figure 1.

Suppose that F_1 represents the composition of the dairy-products portion of the mix, where A is the desired composition. If a line drawn through the points F_1 and A is extended to intersect line CD at G, then G represents the composition of the milk-and-cream mixture which, when combined with F_1 in the proper proportion, will give a mix of the desired composition A. If F_2 represented the composition of the off-batch, then a mixture of cream and evaporated milk would be required for restandardization; whereas if the off-batch were of F_3 composition, a mixture of milk and evaporated milk would be necessary to adjust it to

the desired composition. The method of restandardization in all three cases would be the same; hence only the solution for F_1 will be given.

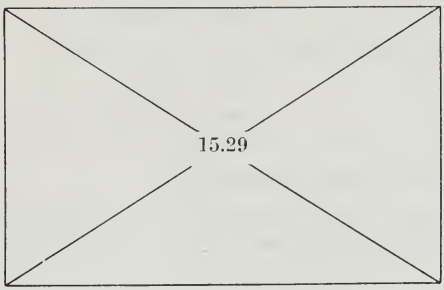
By referring to figure 1 and reading from the graph the values representing the compositions of the ingredients designated by the respective points under consideration, it will be seen that:

F_1, the composition of the dairy products portion of the off-batch, contains 16.50 per cent fat and 13.50 per cent milk-solids-not-fat, which is equivalent to 14.03 per cent fat and 11.48 per cent milk-solids-not-fat for the off-batch as a whole.

A, the desired composition of the dairy products portion of the mix, contains 15.29 per cent fat and 12.35 per cent milk-solids-not-fat, which is equivalent to 13.00 per cent fat and 10.50 per cent milk-solids-not-fat in the final batch.

G, the composition of the milk and cream mixture which is to be combined with the dairy products portion of the off-batch (F_1), contains 10.67 per cent fat and 8.00 per cent milk-solids-not-fat, as can be seen by referring to the graph.

Thus,



4.62 = unit parts of off-batch

1.21 = unit parts of mixture of milk and cream

Then, $4.62 + 1.21 = 5.83$ = total unit parts of mixture testing 15.29 per cent fat,

Hence, $\frac{4.62}{5.83} = 0.79245$ = fraction that the off-batch will be of the final weight of the mix; and,

$\frac{100.00 \text{ pounds}}{0.79245} = 126.19$ pounds = weight of the final batch.

It is more convenient to combine the two previous steps as follows:

$\frac{5.83}{4.62} \times 100$ pounds $= 126.19$ pounds = weight of final batch.

Consequently, $(126.19 - 100.00)$ pounds $= 26.19$ pounds = weight of minimum correcting batch.

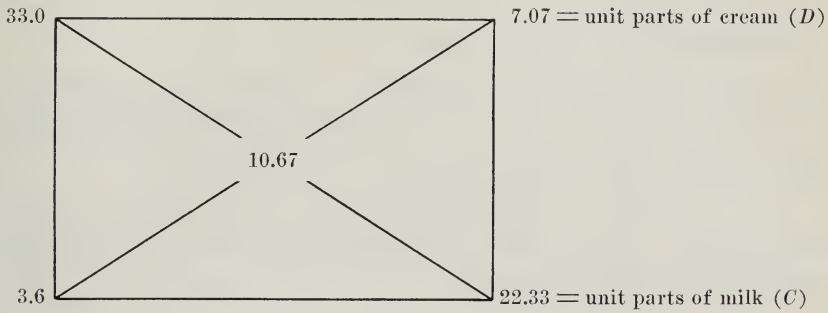
Then, 0.146×26.19 pounds $= 3.82$ pounds of sugar required; and

0.004×26.19 pounds $= 0.10$ pound of gelatin required.

Hence, 26.19 pounds $- (3.82 + 0.10)$ pounds $= 22.27$ pounds = weight of milk-and-cream mixture required.

Next determine the pounds of milk and cream respectively required to make 22.27 pounds of the mixture containing 10.67 per cent fat.

Thus,



Then, $(7.07 + 22.33) = 29.40 =$ total unit parts of milk-and-cream mixture testing 10.67 per cent fat; and

$$\text{the pounds of cream} = \frac{7.07}{29.40} \times 22.27 = 5.36.$$

Therefore, the pounds of milk $= 22.27 - 5.36 = 16.91$.

Tabulated check 8 verifies the above calculations.

TABULATED CHECK 8

Ingredients	Weight	Fat	Milk-solids-not-fat	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Off-batch	100.00	14.03	11.48	40.51
Sugar	3.82			3.82
Gelatin	0.10			0.10
Cream testing 33 per cent fat	5.36	1.77	0.32	2.09
Milk testing 3.6 per cent fat	16.91	0.61	1.47	2.08
Totals supplied	126.19	16.41	13.27	48.60
Totals required	126.19	16.40	13.25	48.57

On a minimum-weight basis, unless the graphic method is employed, it is ordinarily somewhat easier to restandardize a mix containing the fat and milk-solids-not-fat in excess of that desired, than to restandardize one with the fat and serum solids, both too low, or with one of them too low and the other too high. Here the problem consists in getting the fat and milk-solids-not-fat in the desired ratio and then diluting with water to the correct final weight, after considering the required amount of nonmilk products. This can be done by using only one dairy product,

either a fat concentrate, such as cream, or a serum-solids concentrate, such as condensed milk, according to the problem at hand. If the worker desires to use more than one dairy product in any one case, he can then restandardize without the addition of water. Both types of problem will be illustrated in the following sections.

Formula Method of Restandardization.—The following formulas may be used for calculating the amount of cream or condensed milk required for restandardization where the fat and milk-solids-not-fat are both too high and where water will be used for part of the dilution.

First case: Where the fat is not proportionately as high as the milk-solids-not-fat, obviously cream or some other fat concentrate will be necessary for restandardization.

$$\text{Let } R_1 = \frac{\text{per cent fat}}{\text{per cent milk-solids-not-fat}} \text{ in the desired batch;}$$

$$\text{then, pounds of cream} = \frac{\left[\begin{array}{c}R_1 \times \text{pounds} \\ \text{milk-solids-not-fat} \\ \text{in off-batch}\end{array}\right] - \left[\begin{array}{c}\text{pounds} \\ \text{fat in} \\ \text{off-batch}\end{array}\right]}{\left[\begin{array}{c}\text{per cent} \\ \text{fat in} \\ \text{cream}\end{array}\right] - \left[\begin{array}{c}R_1 \times \text{per cent} \\ \text{milk-solids-not-fat} \\ \text{in cream}\end{array}\right]} \times 100.$$

Problem: Suppose 100 pounds of a mix testing 13.0 per cent fat and 41.0 per cent total solids is to be restandardized on a minimum-weight basis to contain 12.0 per cent fat, 10.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.5 per cent gelatin. The products available for restandardization are cream testing 30.0 per cent fat, sugar, gelatin, and water. The gelatin and sugar contents in the original mix are assumed to be correct.

The steps in calculating the solution to this problem are given below:

Calculate the per cent of milk-solids-not-fat in the off-batch by subtracting from the per cent total solids the percentages of fat, sugar, and gelatin. Thus,

$$41.0 - (13.0 + 15.0 + 0.5) = 12.5 = \text{per cent milk-solids-not-fat in the off-batch.}$$

Calculate the ratio of fat to serum solids in the desired mix. Thus,

$$R_1 = \frac{12.0}{10.0} = 1.2.$$

Calculate the pounds of cream required for restandardization by using the formula given above for this case. Thus,

$$\text{pounds cream} = \frac{(1.2 \times 12.5) - 13.0}{30 - (1.2 \times 6.3)} \times 100 = 8.91 \text{ pounds cream.}$$

Calculate the final weight to which the mix must be diluted in either of the following ways:

Divide the total pounds of fat supplied, by the fraction of fat desired in the corrected mix; or, divide the total pounds of milk-solids-not-fat by the fraction of milk-solids-not-fat desired in the corrected batch.

Inasmuch as the method of calculation is the same in both cases, only the first way will be used here. Thus,

$$\begin{aligned} 100 \text{ pounds} \times 0.13 &= 13.00 \text{ pounds fat} \\ 8.91 \text{ pounds} \times 0.30 &= 2.67 \text{ pounds fat} \\ \text{Total supplied} &= 15.67 \text{ pounds fat} \end{aligned}$$

$$\text{Hence, } \frac{15.67 \text{ pounds}}{0.12} = 130.58 \text{ pounds} = \text{final weight of mix.}$$

The additional weights of sugar and gelatin required are calculated in the usual manner. Thus,

$$(130.58 - 100.00) \text{ pounds} = 30.58 \text{ pounds} = \text{weight of correcting batch.}$$

$$\begin{aligned} \text{Then, } 30.58 \text{ pounds} \times 0.15 &= 4.59 \text{ pounds sugar; and} \\ 30.58 \text{ pounds} \times 0.005 &= 0.15 \text{ pound gelatin.} \end{aligned}$$

The weight of water for the final dilution equals the difference between the total weight desired and the weight of the ingredients required to supply the necessary solids in the mix. Thus,

$$30.58 \text{ pounds} - (8.91 + 4.59 + 0.15) \text{ pounds} = 16.93 \text{ pounds of water.}$$

These calculations are verified in tabulated check 9.

TABULATED CHECK 9

Ingredients	Weight	Fat	Milk-solids-not-fat	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Off-batch testing 13.0 per cent fat and 41.0 per cent total solids	100.00	13.00	12.50	41.00
Cream testing 30.0 per cent fat	8.91	2.67	0.56	3.23
Sugar	4.59			4.59
Gelatin	0.15			0.15
Water	16.93			
Totals supplied	130.58	15.67	13.06	48.97
Totals required	130.58	15.67	13.06	48.97

Second case: Where the milk-solids-not-fat are not proportionally as high as the fat, condensed milk or some other serum-solids concentrate will be necessary to adjust the mix to the desired composition.

$$\text{Let } R_2 = \frac{\text{per cent milk-solids-not-fat}}{\text{per cent fat}} \text{ in the desired batch.}$$

$$\text{Then the pounds of condensed milk} = \frac{\left[\begin{array}{c}R_2 \times \text{pounds}\\ \text{fat in}\\ \text{off-batch}\end{array}\right] - \left[\begin{array}{c}\text{pounds milk-}\\ \text{solids-not-fat}\\ \text{in off-batch}\end{array}\right]}{\left[\begin{array}{c}\text{per cent milk-}\\ \text{solids-not-fat}\\ \text{in condensed}\\ \text{milk}\end{array}\right] - \left[\begin{array}{c}R_2 \times \text{per cent}\\ \text{fat in}\\ \text{condensed}\\ \text{milk}\end{array}\right]} \times 100.$$

Problem: Suppose 100 pounds of a mix testing 15.0 per cent fat and 41.0 per cent total solids is to be restandardized on a minimum-weight basis to contain 12.5 per cent fat, 10.0 per cent milk-solids-not-fat, 14.5 per cent sugar, and 0.5 per cent gelatin. The gelatin and sugar contents in the original batch are assumed to be correct. The milk product available for restandardization is evaporated milk testing 8.0 per cent fat and 20.0 per cent milk-solids-not-fat. The steps in the restandardization are as follows:

Calculate the per cent milk-solids-not-fat in the off-batch by difference as in the first case (page 30). Thus,

$$41.0 \text{ per cent} - (15.0 + 14.5 + 0.5) \text{ per cent} = 11.0 \text{ per cent.}$$

The ratio of milk-solids-not-fat to fat is calculated for the desired mix. Thus,

$$R_2 = \frac{10.0}{12.5} = 0.80.$$

Calculate the pounds of evaporated milk required by using the formula given for this case. Thus,

$$\text{pounds evaporated milk} = \frac{(0.80 \times 15.0) - 11.0}{20.0 - (0.80 \times 8.0)} \times 100 = 7.35 \text{ pounds.}$$

Ascertain the final weight to which the mix must be diluted by either of the methods suggested in the first case. The calculations here will be made on the basis of the fat content. Thus,

$$\begin{aligned} 100 \quad &\text{pounds} \times 0.15 = 15.00 \text{ pounds fat} \\ 7.35 \text{ pounds} &\times 0.08 = \underline{\quad 0.59 \text{ pound fat}} \\ &\text{Total fat in mix} = 15.59 \text{ pounds fat} \end{aligned}$$

$$\text{Hence, final weight of mix} = \frac{15.59 \text{ pounds}}{0.125} = 124.72 \text{ pounds.}$$

The additional weights of sugar and gelatine required are calculated as usual. Thus,

(124.72 — 100.00) pounds = 24.72 pounds = weight of correcting batch.
Then, 24.72 pounds × 0.145 = 3.58 pounds sugar; and
24.72 pounds × 0.005 = 0.12 pound gelatin.

The weight of water for the final dilution is calculated by difference, as in the first case. Thus,

24.72 pounds — (7.35 + 3.58 + 0.12) pounds = 13.67 pounds = weight of water required.

Tabulated check 10 verifies the preceding calculations.

TABULATED CHECK 10

Ingredients	Weight	Fat	Serum solids	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Off-batch testing 15.0 per cent fat and 41.0 per cent total solids	100.00	15.00	11 00	41.00
Evaporated milk testing 8.0 per cent fat and 20.0 per cent milk-solids-not-fat	7.35	0.59	1 47	2.06
Sugar	3.58			3.58
Gelatin	0 12			0.12
Water	13.67			
Totals supplied	124.72	15 59	12.47	46.76
Totals required	124.72	15.59	12.47	46.77

Algebraic Method of Restandardization.—When the algebraic method is used for restandardization on a minimum-weight basis without dilution with water, the principle is the same whether both fat and serum solids in the off-batch are too high or too low, or whether one is too high and the other too low. The first step is to determine which of the dairy products available will be required for restandardization. Then set up simultaneous equations using the unknowns and solve as previously indicated.

As an illustration of this method, suppose it is desired to restandardize 100 pounds of mix testing 11.0 per cent fat, 10.0 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.4 per cent gelatin so that the final batch will contain 13.0 per cent fat, 10.5 per cent milk-solids-not-fat, 15.0 per cent sugar, and 0.4 per cent gelatin. The dairy products available are cream testing 40.0 per cent fat, milk testing 4.0 per cent fat, and condensed skim milk testing 32.0 per cent serum solids.

Obviously, in this case, only cream and condensed skim milk will be required for the restandardization, as far as dairy products are concerned. Just which products will be required for the restandardization is not always so apparent, although by use of the graph illustrated in figure 1, this can easily be determined in all cases. The unknowns are as follows:

X = weight of cream required
Y = weight of condensed skim milk required
Z = minimum weight of the final batch

The equations will then be

(a) $0.846Z = X + Y + 84.60$ (milk products equation)
(b) $0.105Z = 0.054X + 0.32Y + 10.00$ (milk-solids-not-fat equation)
(c) $0.130Z = 0.40X + 11.00$ (fat equation)

The solution follows; subtracting,

(d)	$27.072Z = 32X + 32Y + 2{,}707.2$		(Equation $a \times 32$)
(e)	$10.500Z = 5.4X + 32Y + 1{,}000.0$		(Equation $b \times 100$)
(f)	$16.572Z = 26.60X +$	$1{,}707.2$	(Equation d — equation e)
(g)	$0.623Z = X +$	64.18	(Equation $f \div 26.6$)
(h)	$0.325Z = X +$	27.50	(Equation $c \div 0.4$)
(i)	$0.298Z =$	36.68	(Equation g — equation h)

Solving equation (i) for Z

$$Z = \frac{36.68}{0.298} = 123.09.$$

Substitute this value of Z in equation (c) and solve for X:

$$0.13 \times 123.09 = 0.4X + 11.0$$

$$X = \frac{16.00 - 11.00}{0.4} = 12.50.$$

Substitute these values in equation (a) and solve for Y:

$$0.846 \times 123.09 = 12.50 + Y + 84.60$$
$$Y = 104.13 - (84.60 + 12.50) = 7.03.$$

The nonmilk products required are calculated as usual. Thus,

(123.09 — 100.00) pounds = 23.09 pounds = weight of correcting batch.
Then, 23.09 pounds × 0.15 = 3.46 pounds = weight of sugar; and
23.09 pounds × 0.004 = 0.09 pound = weight of gelatin.

Tabulated check 11 verifies these calculations.

TABULATED CHECK 11

Ingredients	Weight	Fat	Milk-solids-not-fat	Total solids
	pounds	*pounds*	*pounds*	*pounds*
Off-batch testing 11.0 per cent fat and 36.4 per cent total solids	100.00	11.00	10.00	36.40
Cream testing 40.0 per cent fat	12.50	5.00	0.68	5.68
Condensed skim milk testing 32.0 per cent milk-solids-not-fat	7.03		2.25	2.25
Sugar	3.46			3.46
Gelatin	0.09			0.09
Totals supplied	123.08	16.00	12.93	47.88
Totals required	123.09	16.00	12.92	47.88

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